Functions

Limits (part 1)

Definition

➢A limit tells us the value **b** that a function approaches as that function's inputs (x) get closer and closer to $\boldsymbol{a}$ (a can be a number or infinity).

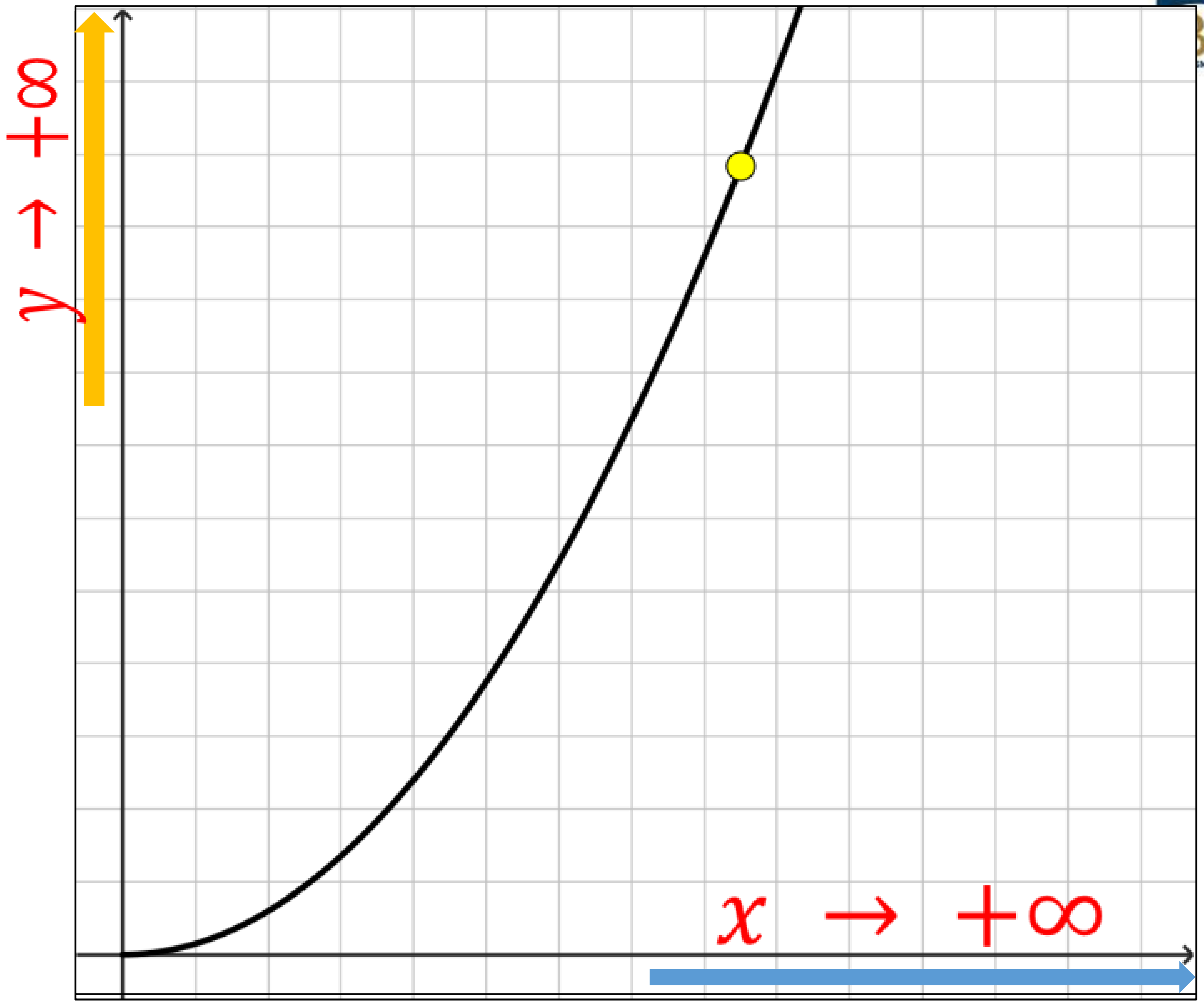
We say that : $f(x)$ tends to b when x tends to a

$$\left.\begin{matrix} \boldsymbol{f(x) \to b} \\ \boldsymbol{x \to a} \end{matrix}\right\} \quad \lim_{x \to a} f(x) = b$$

Limit at infinity

Example 1

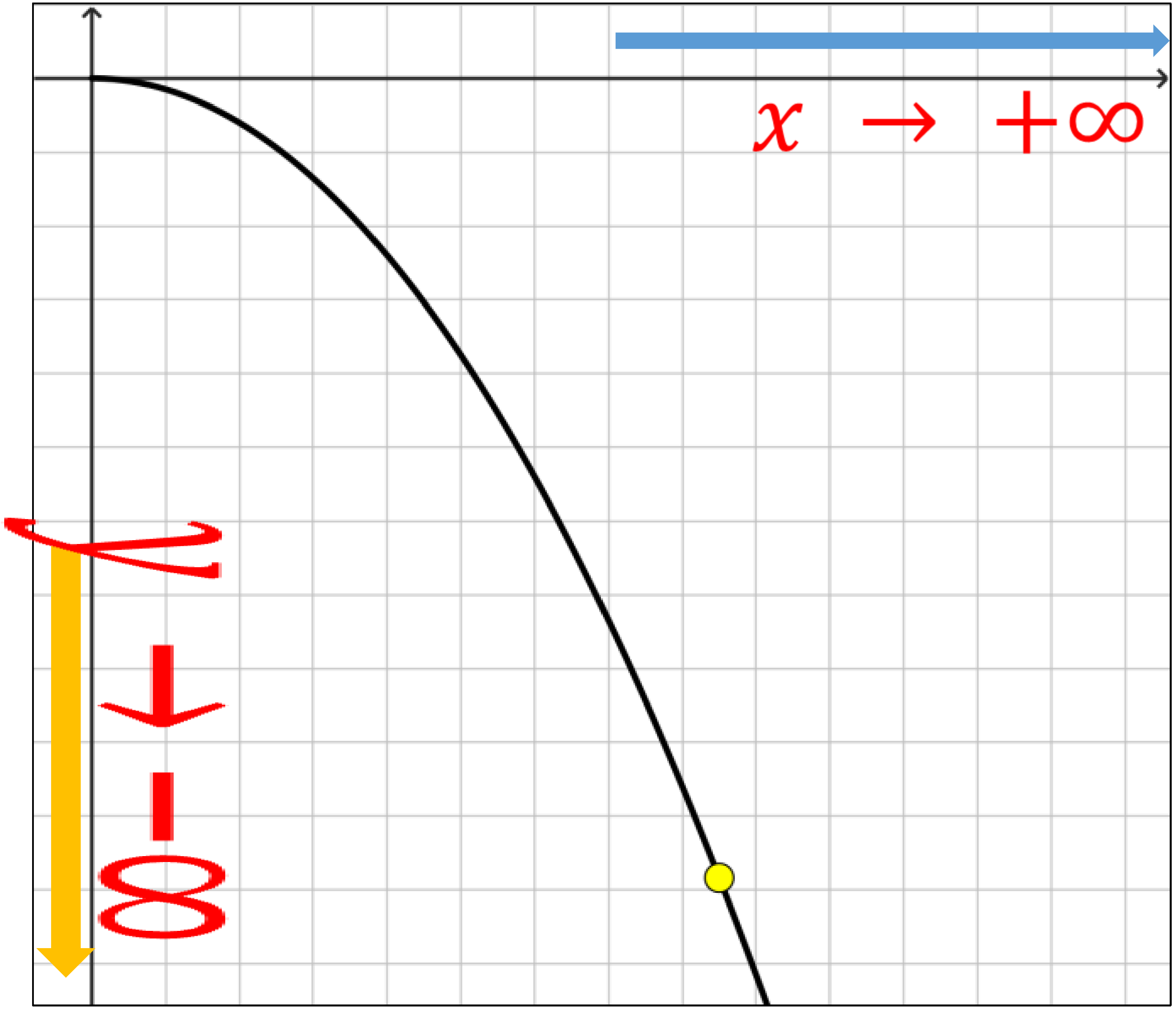
$$\lim_{x \to +\infty} f(x) = +\infty$$



Limit at infinity

Example 2

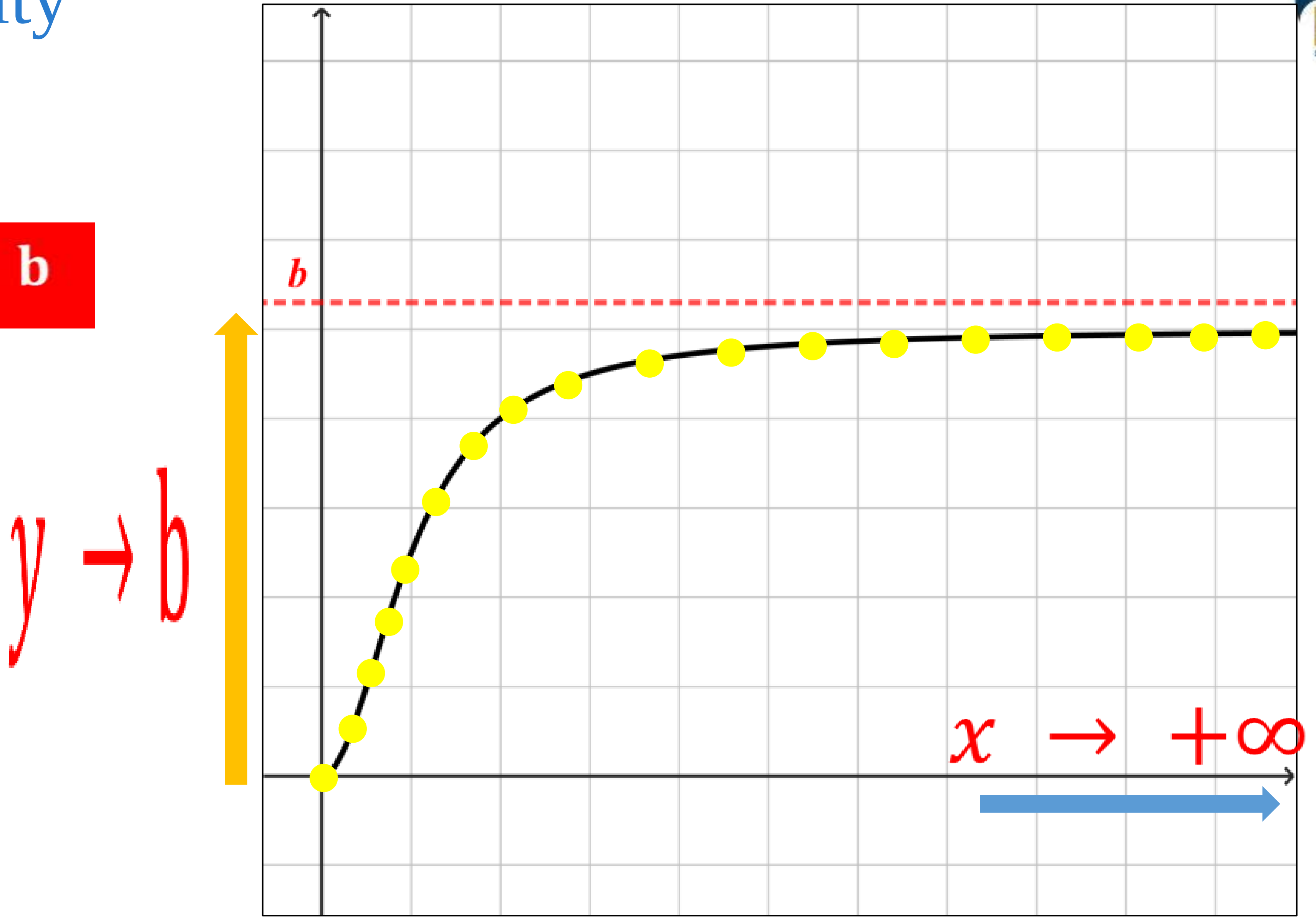
$$\lim_{x \to +\infty} f(x) = -\infty$$



Limit at infinity

Example 3

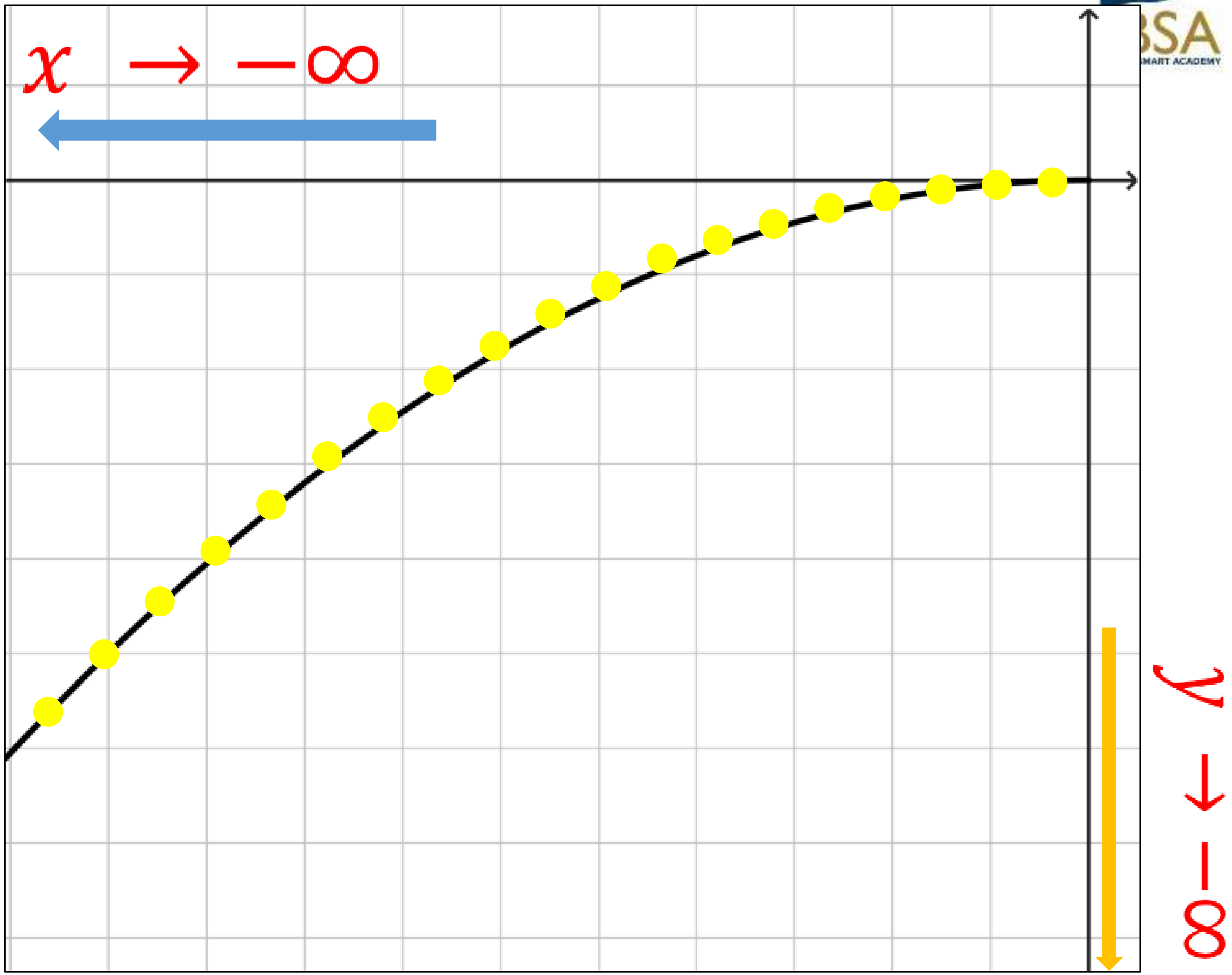
$$\lim_{x \to +\infty} f(x) = b$$



Limit at infinity

Example 4

$$\lim_{x \to -\infty} f(x) = -\infty$$



Limit at infinity

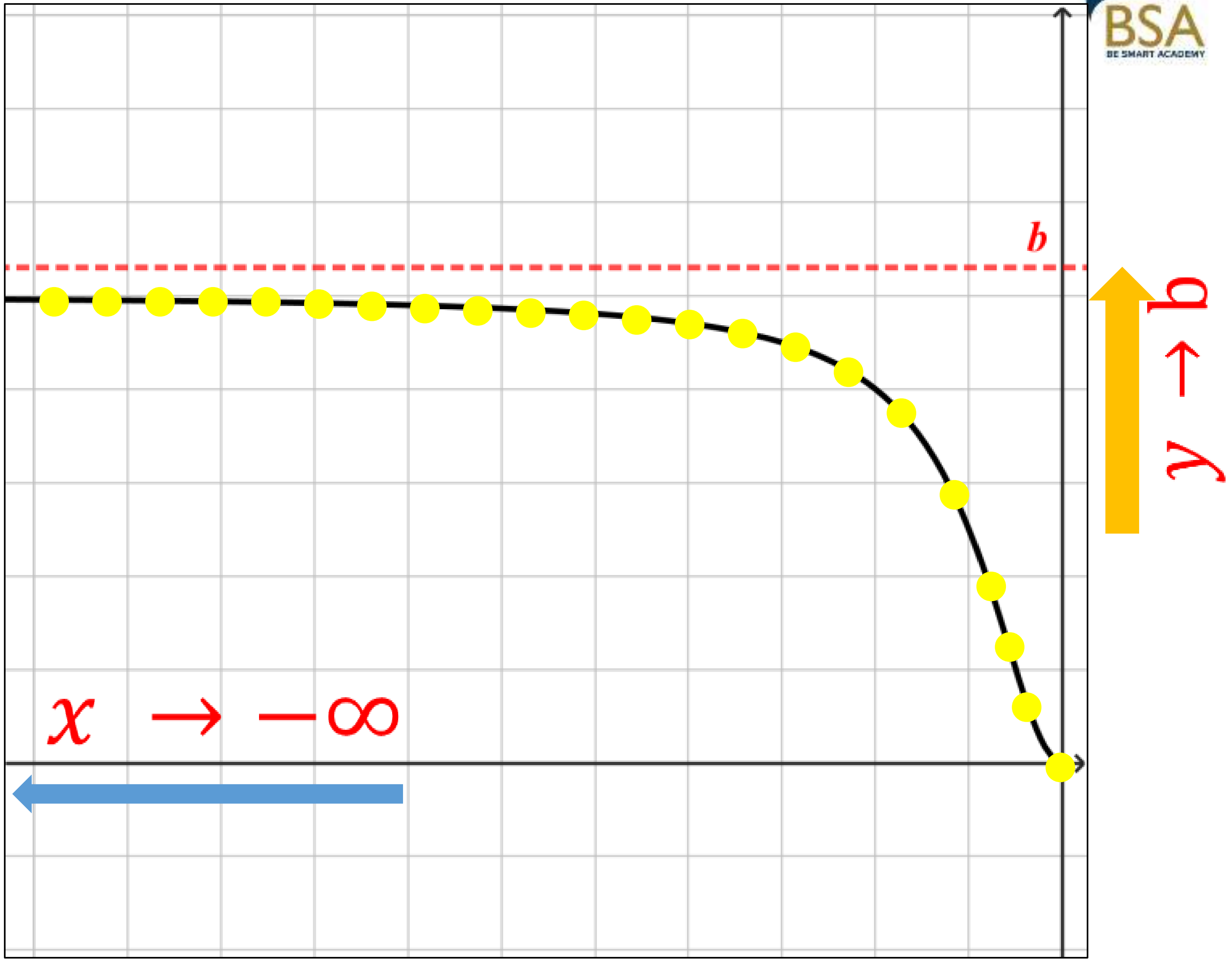
Example 5

$$\lim_{x \to -\infty} f(x) = +\infty$$

Limit at infinity

Example 6

$$\lim_{x \to -\infty} f(x) = b$$

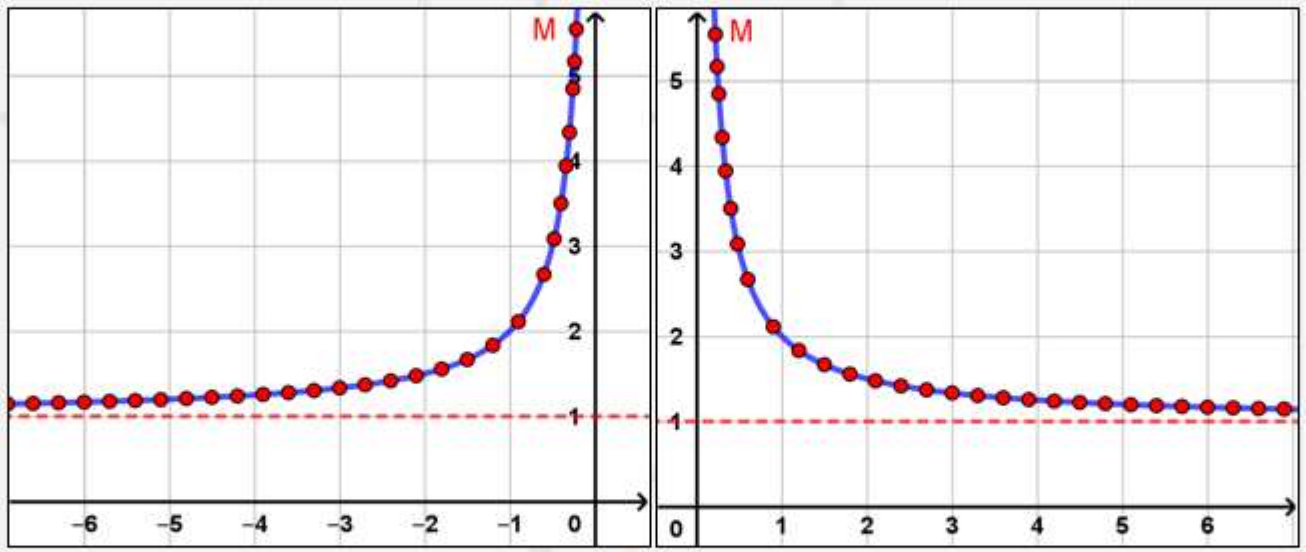


Horizontal asymptote

As x increases indefinitely to $+\infty$ or decreases indefinitely to $-\infty$, $f(x)$ can approaches toward a horizontal line of equation y=b.

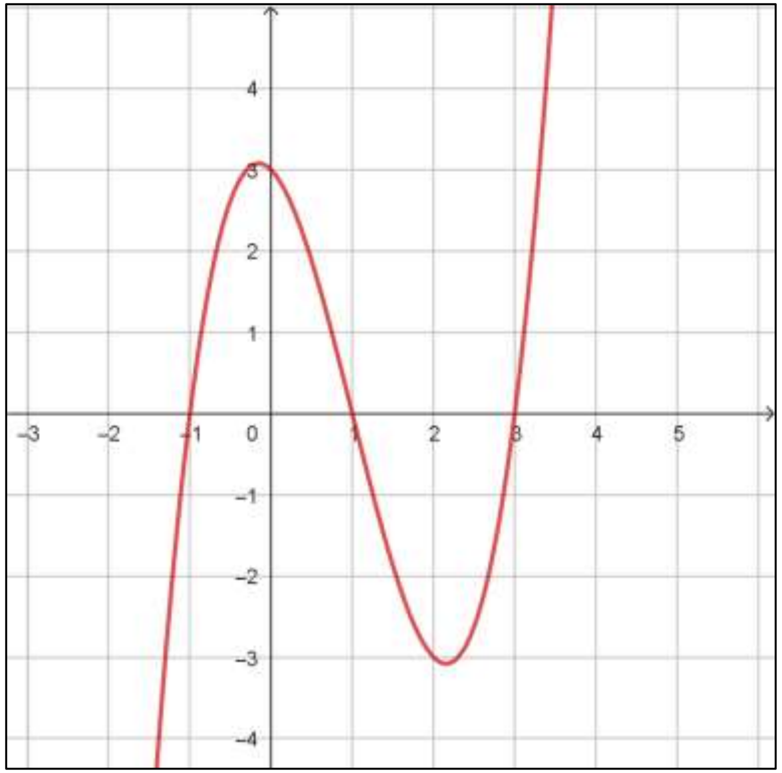
This line is called:

horizontal asymptote



Application

Find the limits at infinity in each case and determine the horizontal asymptote if exists.

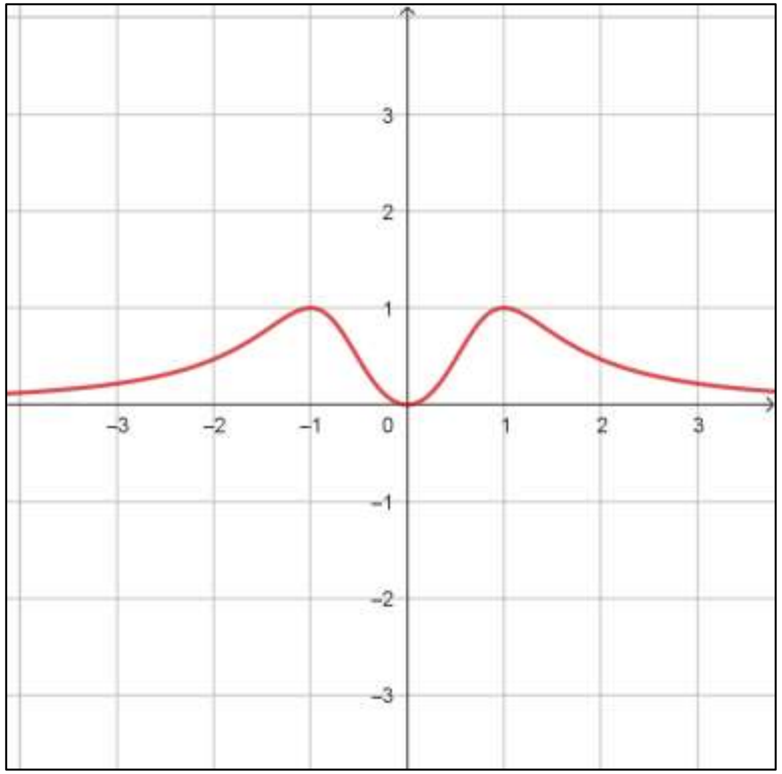


$$\lim_{x \to +\infty} f(x) = +\infty$$

$$\lim_{x \to -\infty} f(x) = -\infty$$

Application

Find the limits at infinity in each case and determine the horizontal asymptote if exists.



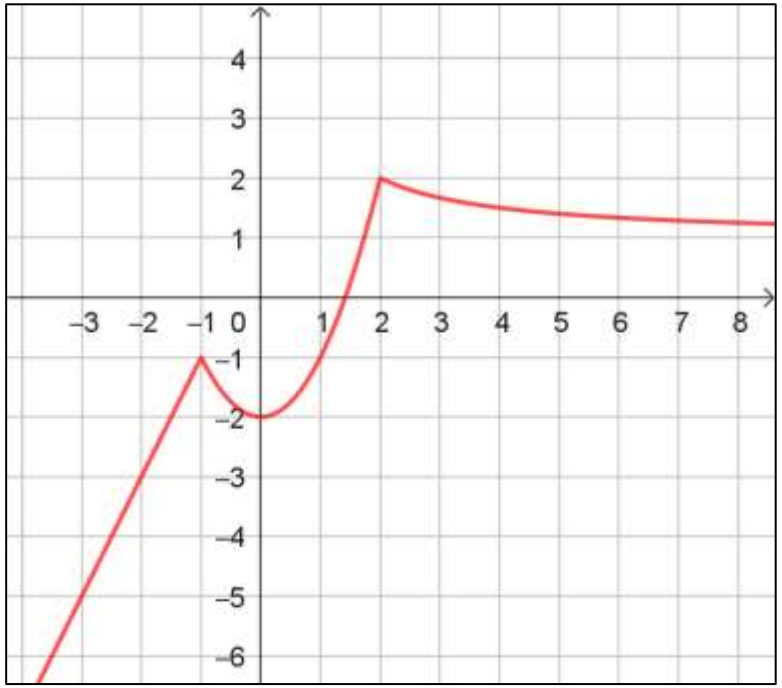
$$\lim_{x \to +\infty} f(x) = 0$$

$$\lim_{x \to -\infty} f(x) = 0$$

(x'x) is a horizontal asymptote near to $\pm\infty$

Application

Find the limits at infinity in each case and determine the horizontal asymptote if exists.



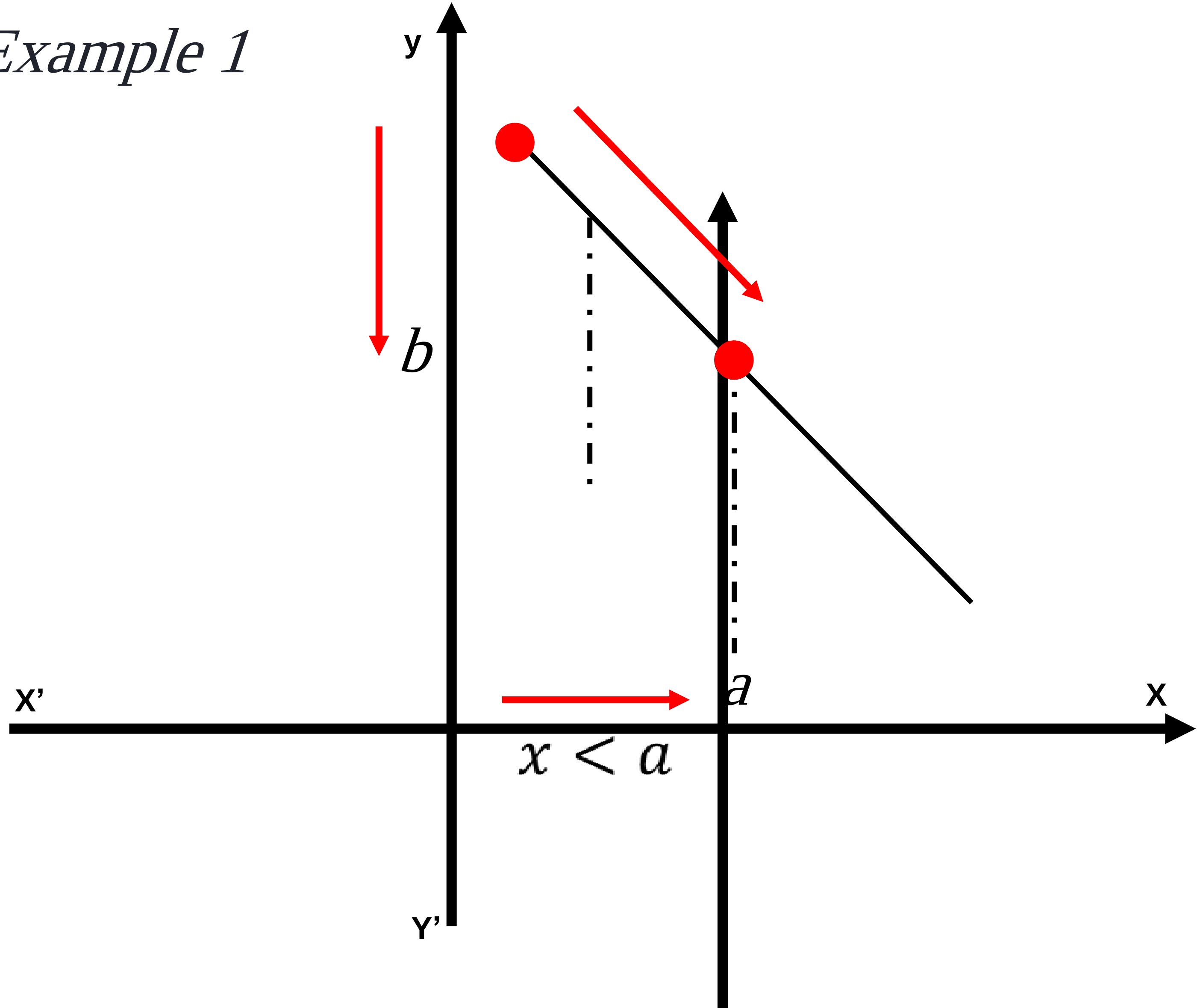
$$\lim_{x \to +\infty} f(x) = 1$$

y=1 is a horizontal asymptote near to +∞

$$\lim_{x \to -\infty} f(x) = -\infty$$

Limit at a point

Example 1



Limit from below is b

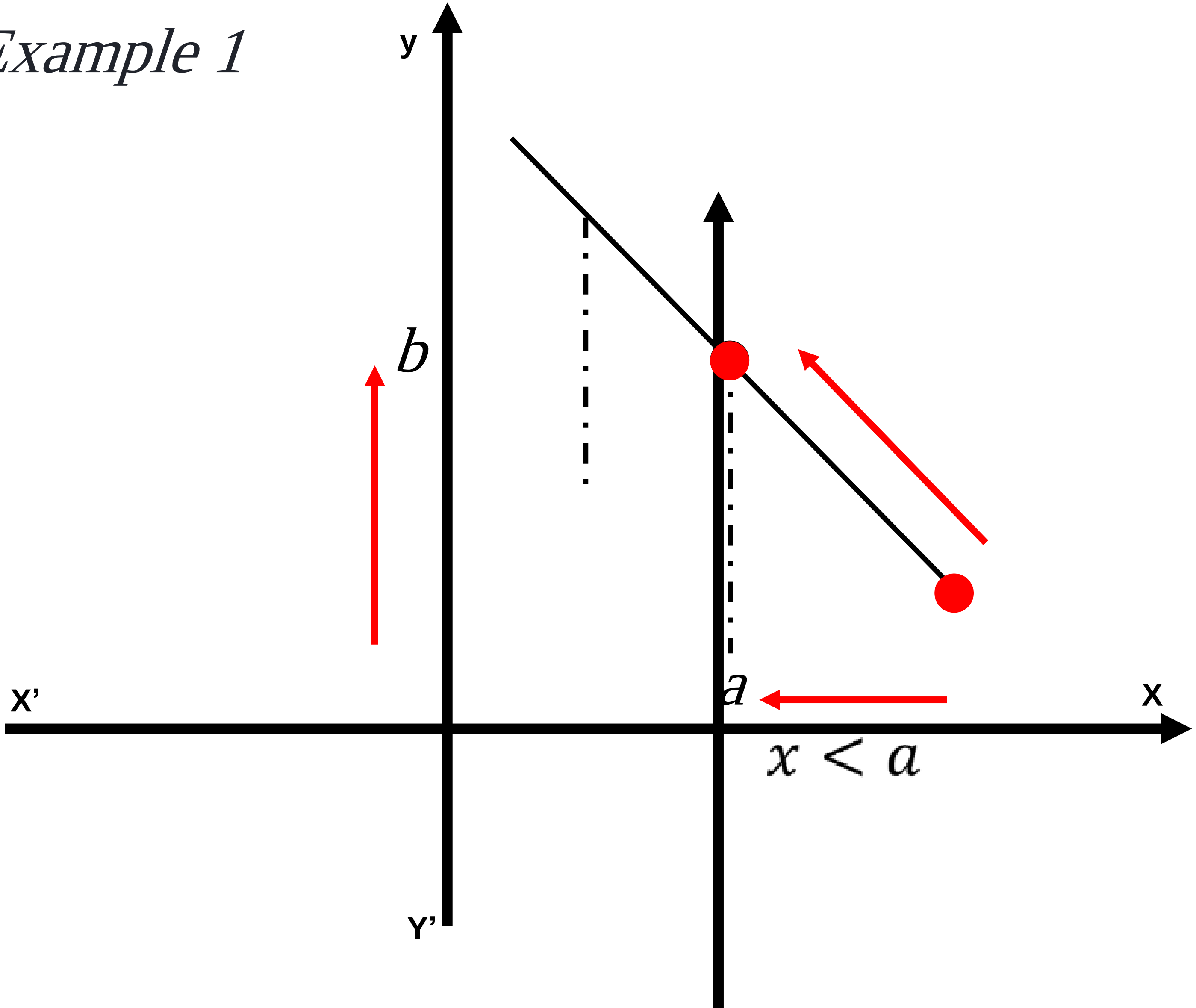
$$\lim_{\substack{x\to a \\ x<a}} f(x) = b$$

or

$$\lim_{x\to a^-} f(x) = b$$

Limit at a point

Example 1



Limit from above is b

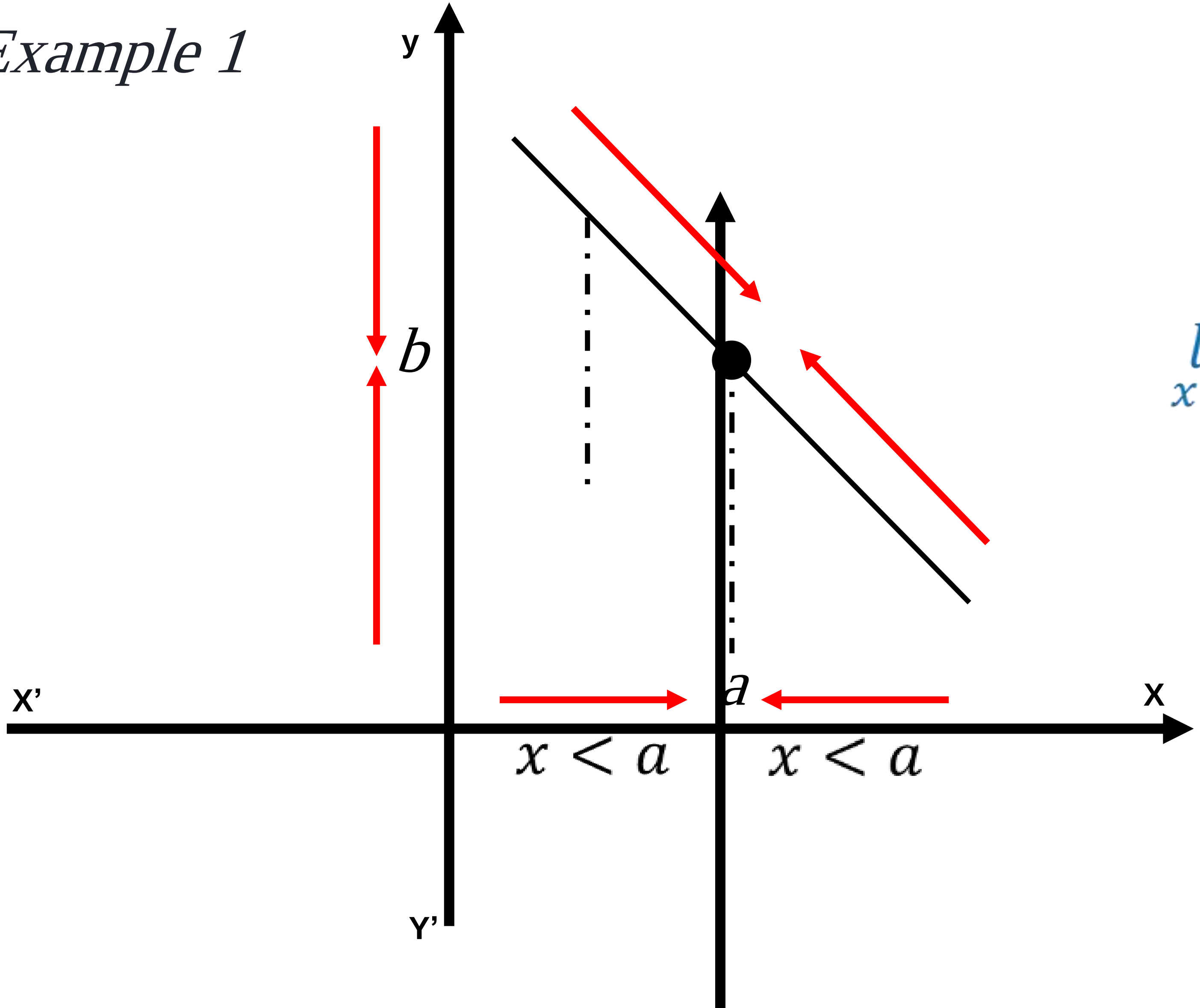
$$\lim_{\substack{x \to a \\ x > a}} f(x) = b$$

or

$$\lim_{x \to a^+} f(x) = b$$

Limit at a point

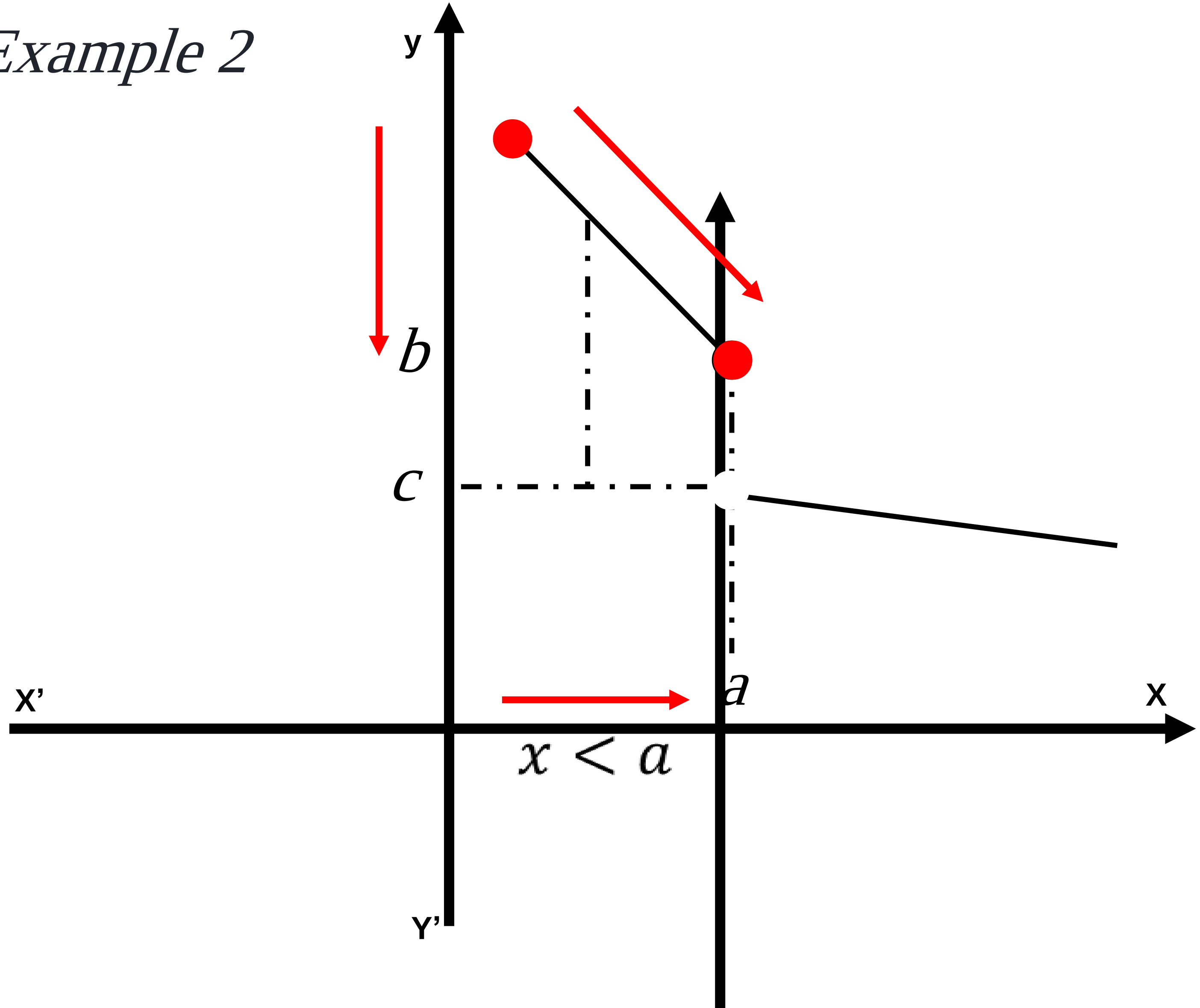
Example 1



$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

Limit at a point

Example 2



Limit from below is b

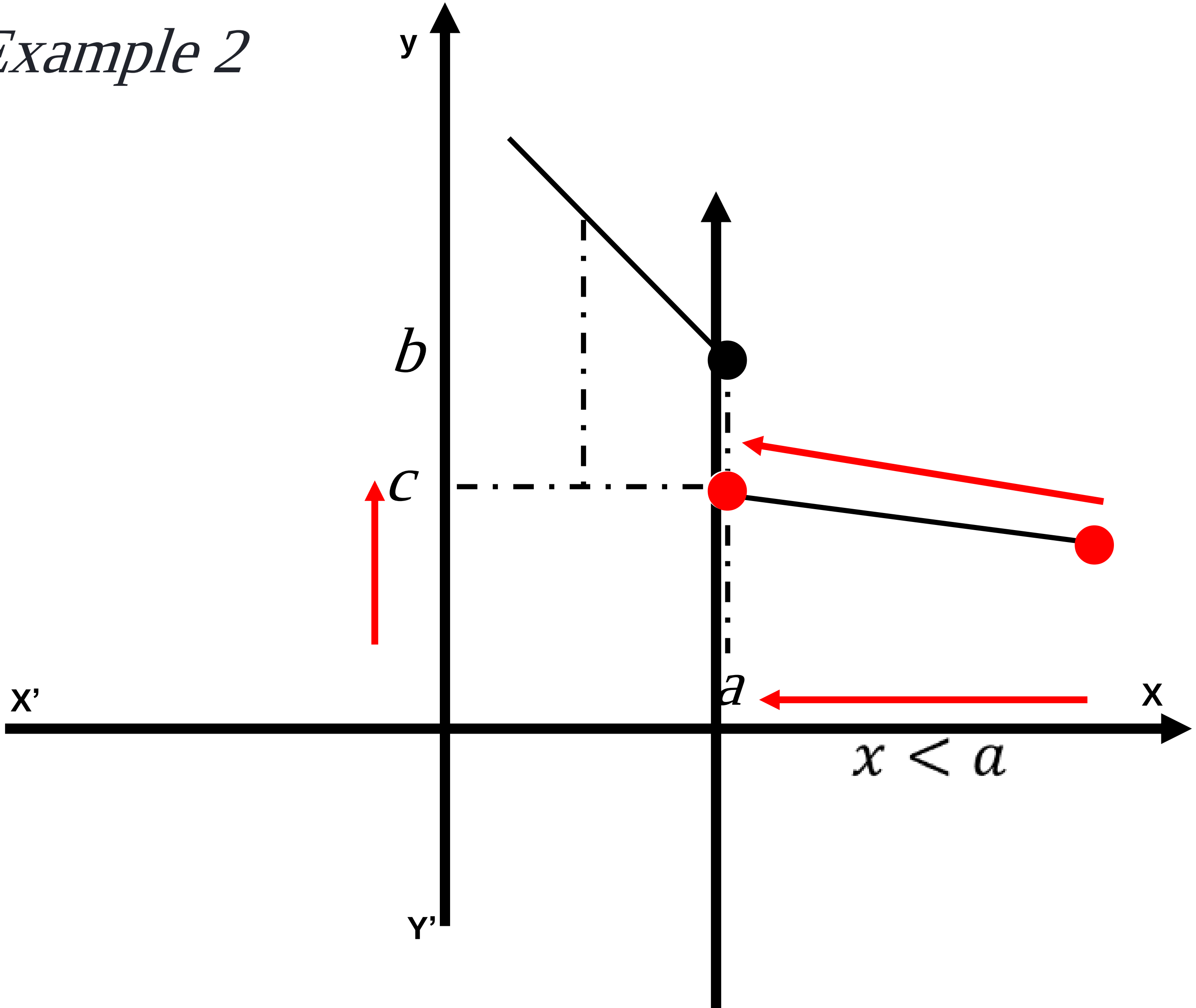
$$\lim_{\substack{x\to a \\ x<a}} f(x) = b$$

or

$$\lim_{x\to a^-} f(x) = b$$

Limit at a point

Example 2



Limit from above is c

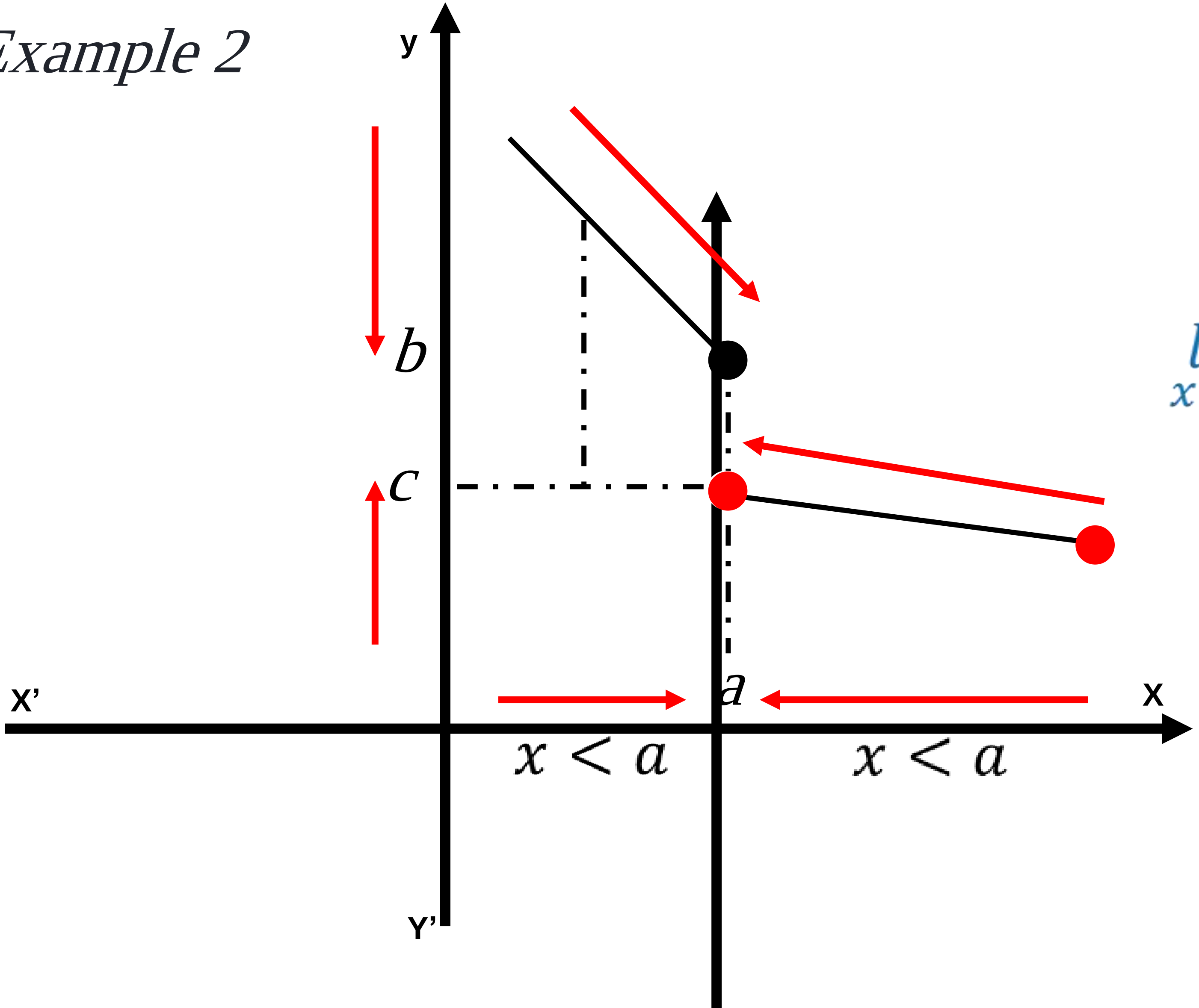
$$\lim_{\substack{x \to a \\ x > a}} f(x) = c$$

or

$$\lim_{x \to a^+} f(x) = c$$

Limit at a point

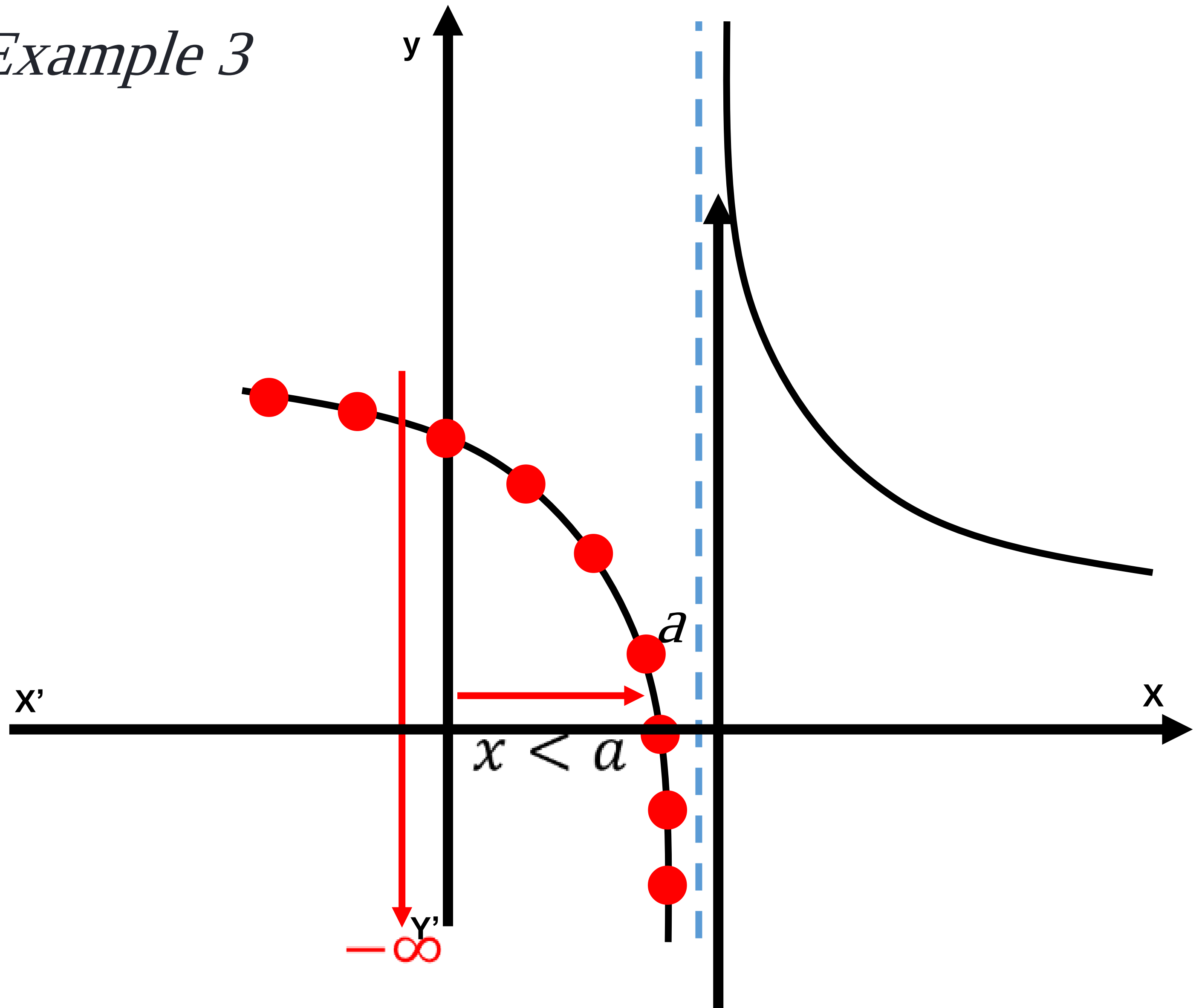
Example 2



$$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$$

Limit at a point

Example 3



Limit from below is -∞

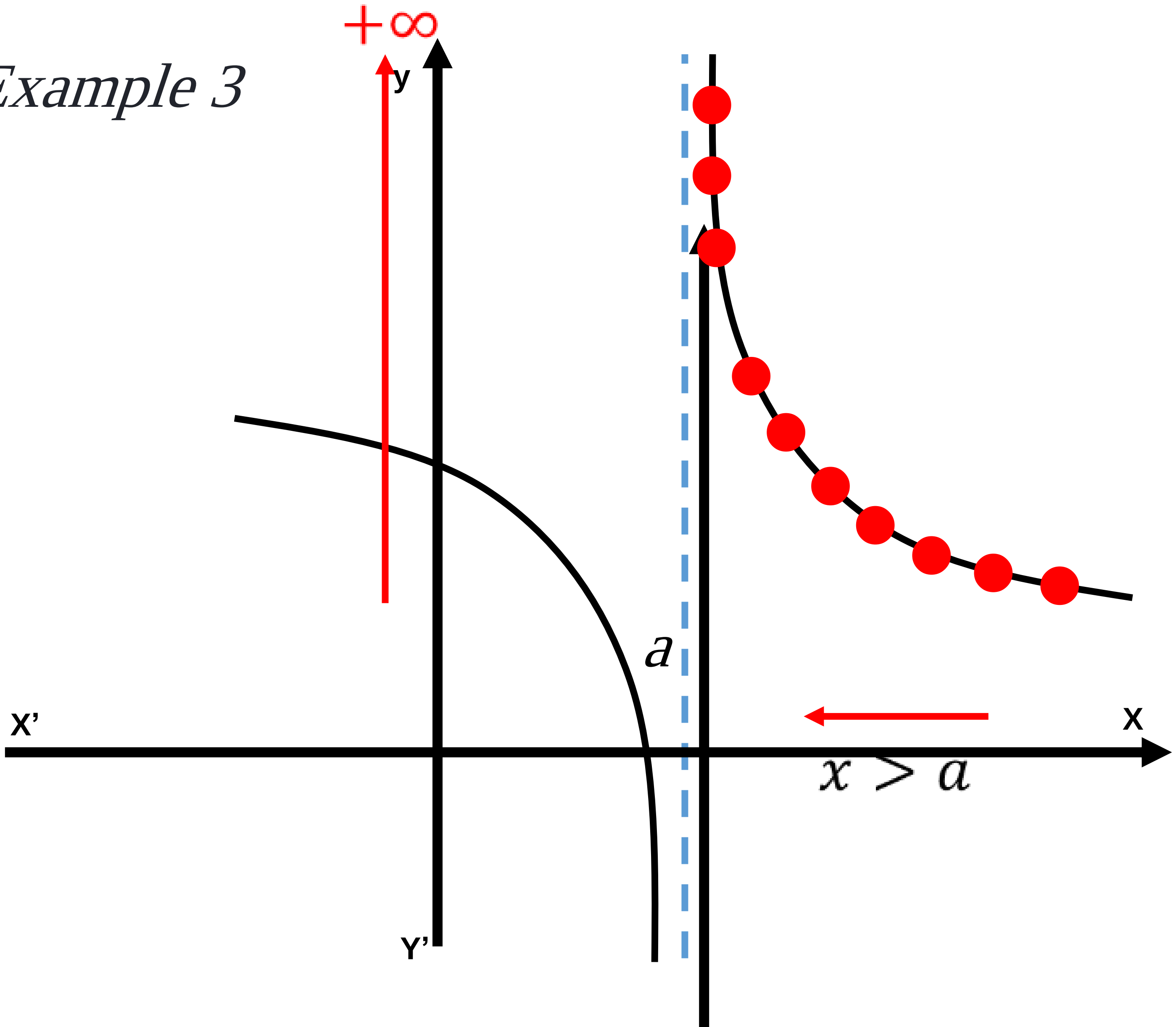
$$\lim_{\substack{x \to a \\ x < a}} f(x) = -\infty$$

or

$$\lim_{x \to a^-} f(x) = -\infty$$

Limit at a point

Example 3



Limit from above is +∞

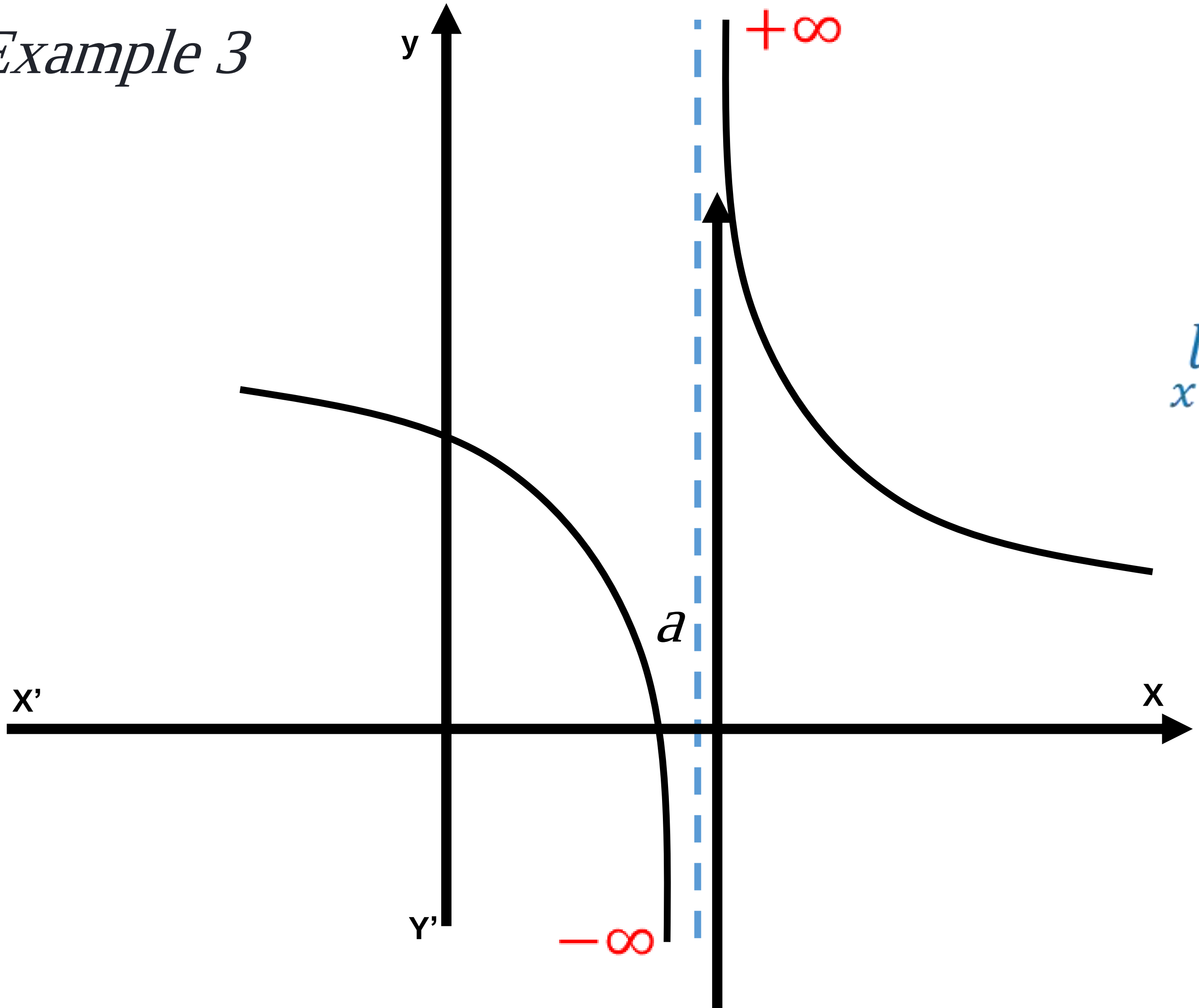
$$\lim_{\substack{x \to a \\ x > a}} f(x) = +\infty$$

or

$$\lim_{x \to a^+} f(x) = +\infty$$

Limit at a point

Example 3

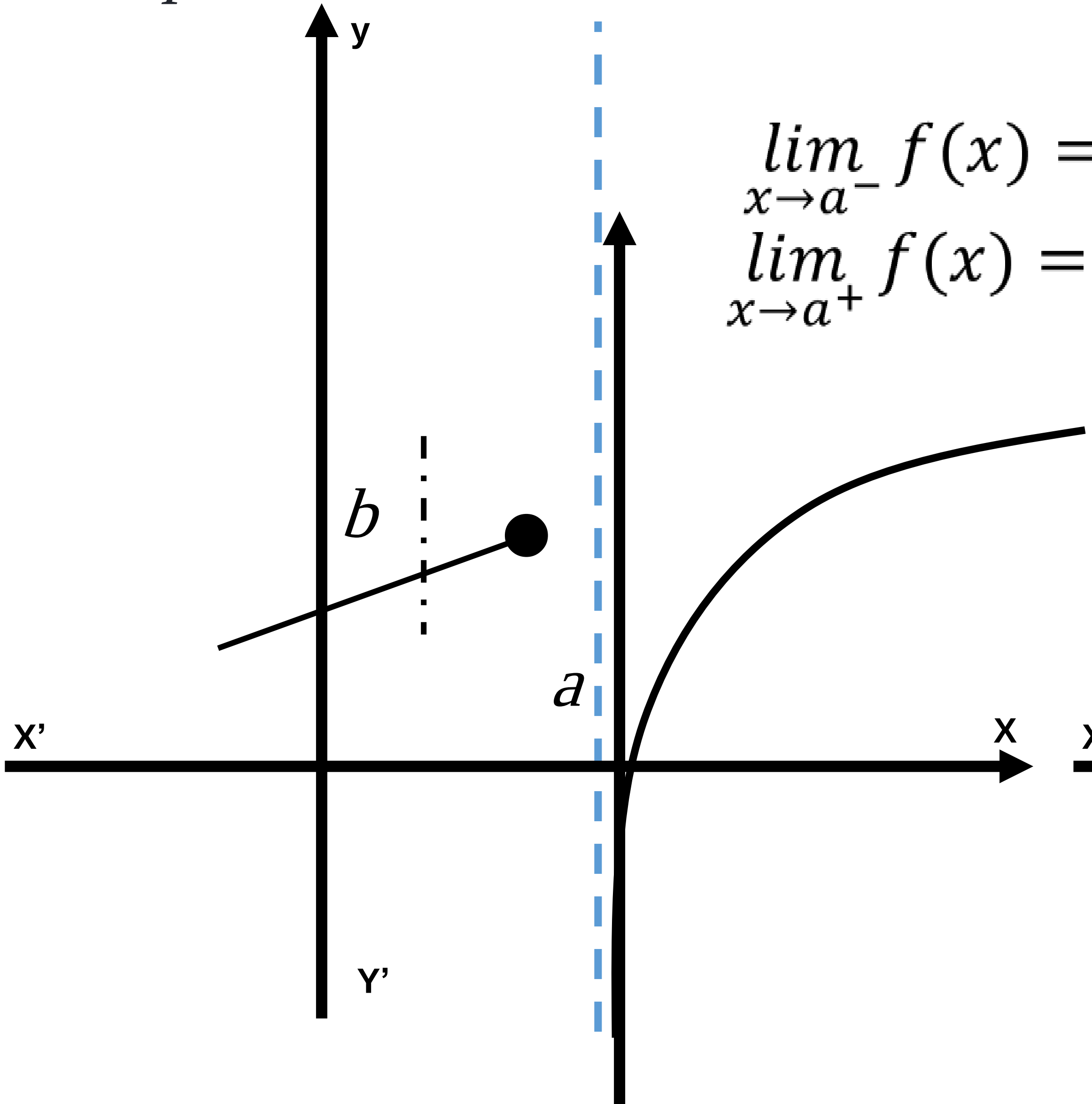


$$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$$

Limit at a point

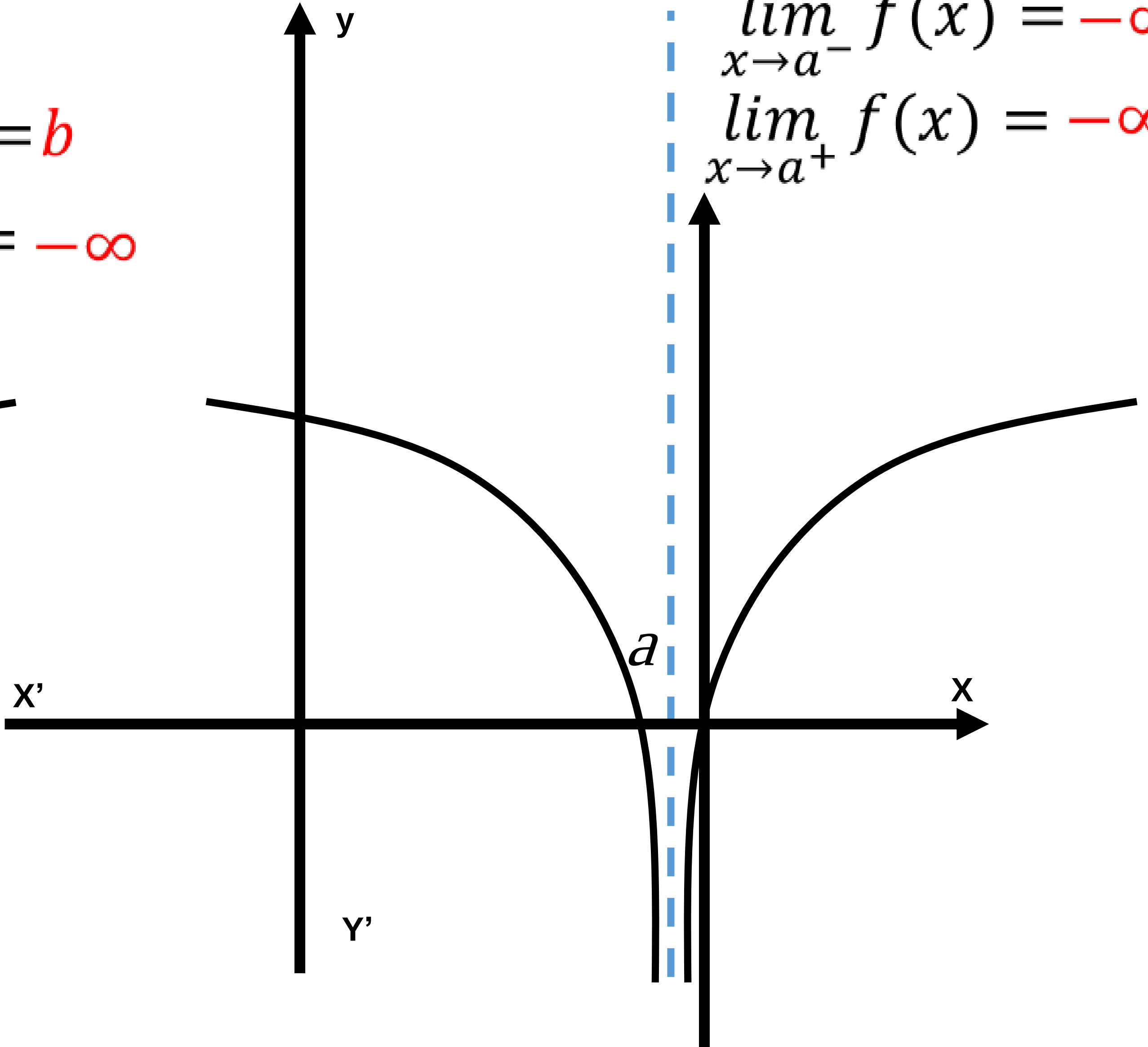
Example 4

Can you find the given limits by yourself?



$$\lim_{x \to a^-} f(x) = b$$

$$\lim_{x \to a^+} f(x) = -\infty$$



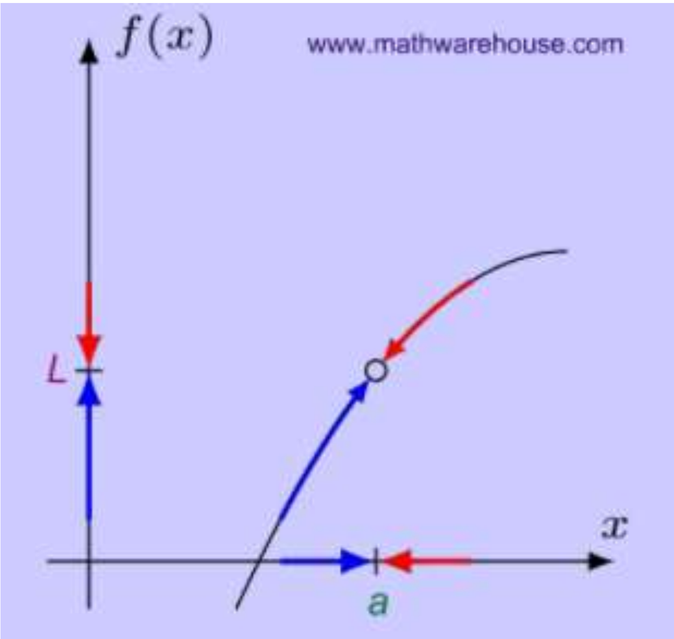
$$\lim_{x \to a^-} f(x) = -\infty$$

$$\lim_{x \to a^+} f(x) = -\infty$$

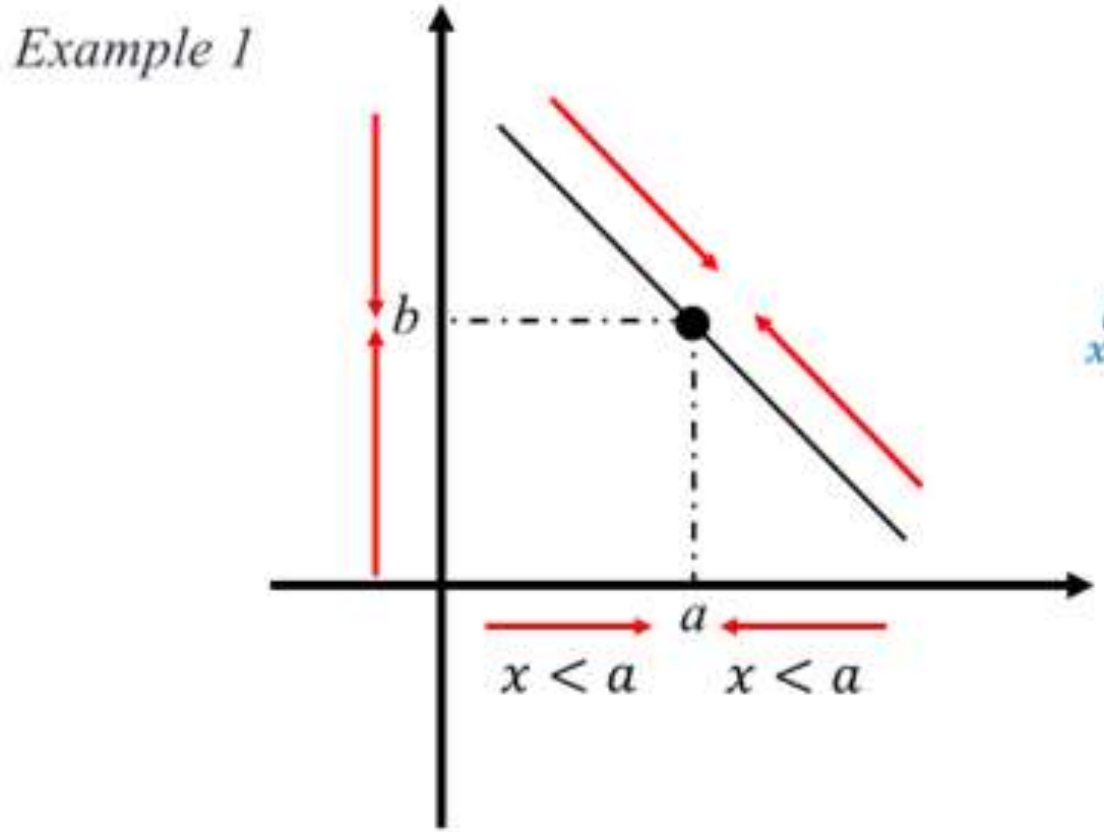
Limit at a point

The limit at a point exist if the limit from above is equal to the limit from below:

$$\lim_{x \to a} f(x) = \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$$



Limit at a point

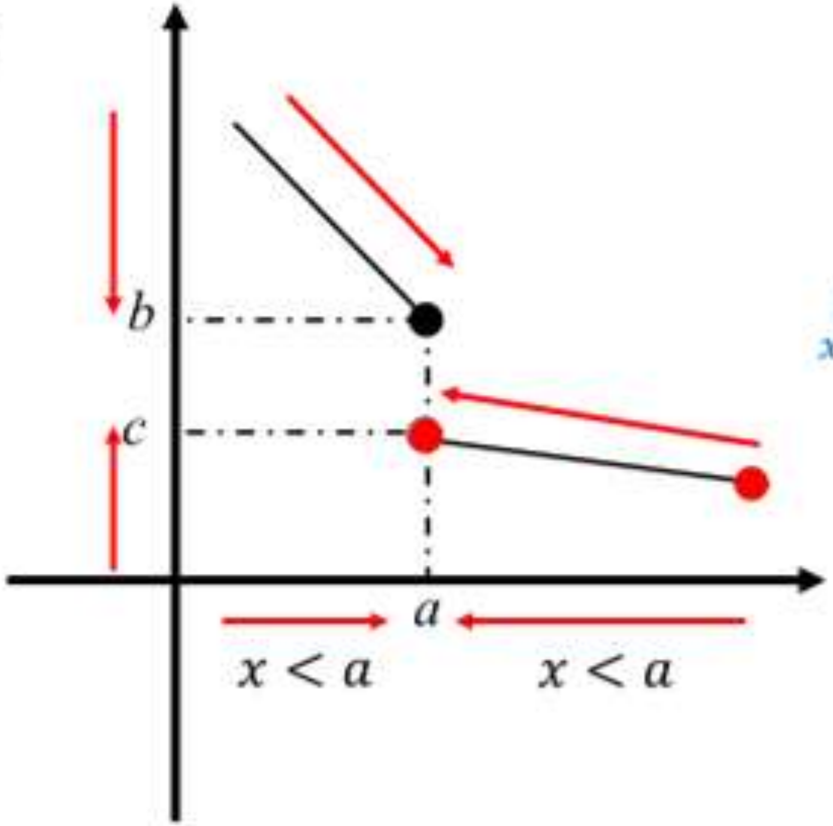


$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

$$\lim_{x \to a} f(x) = b$$

Limit at a point

Example 2

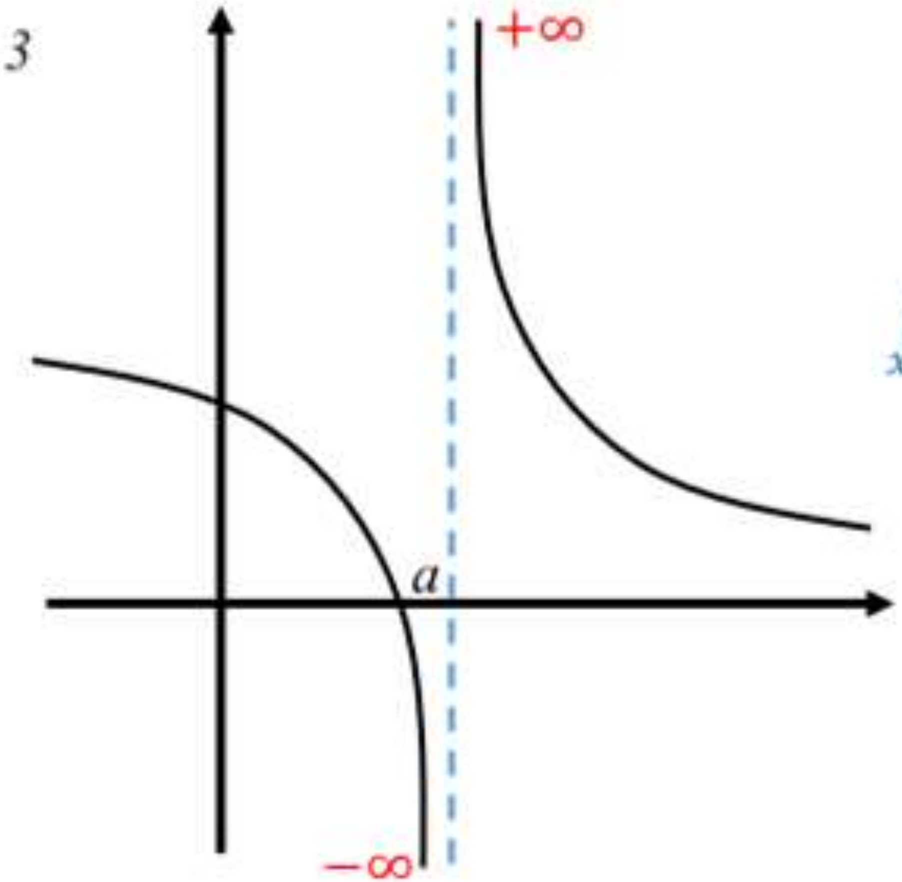


$$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$$

$$\lim_{x \to a} f(x) = \nexists$$

Limit at a point

Example 3

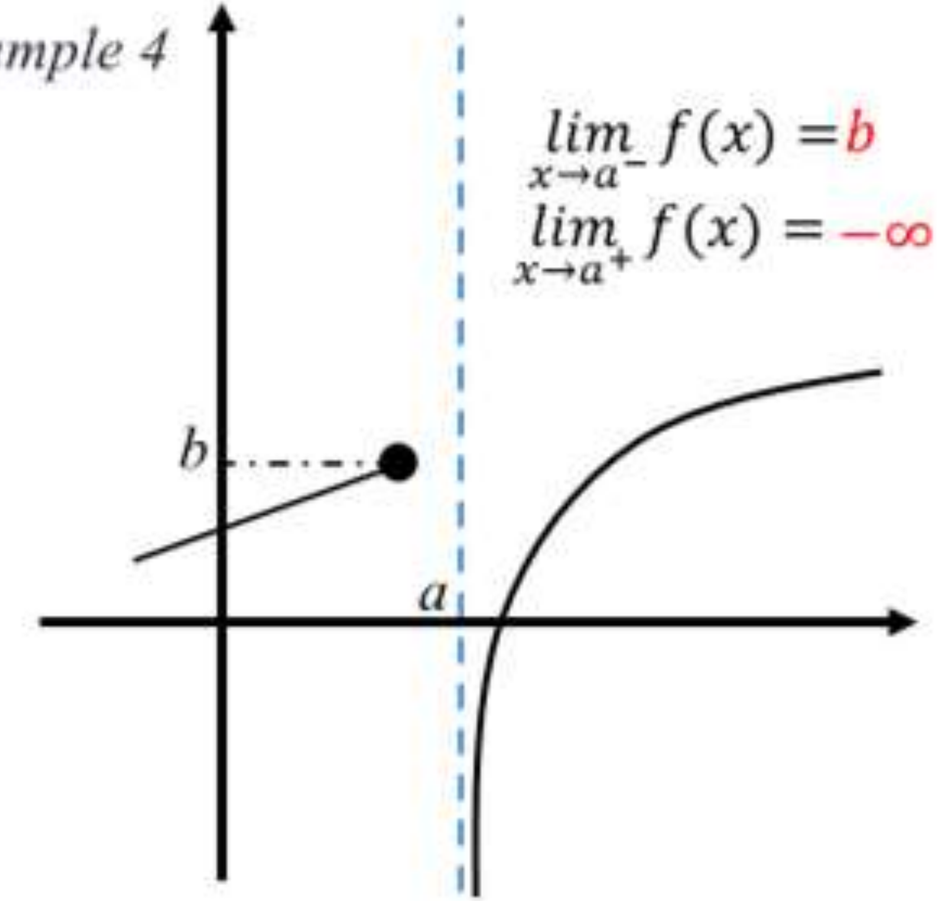


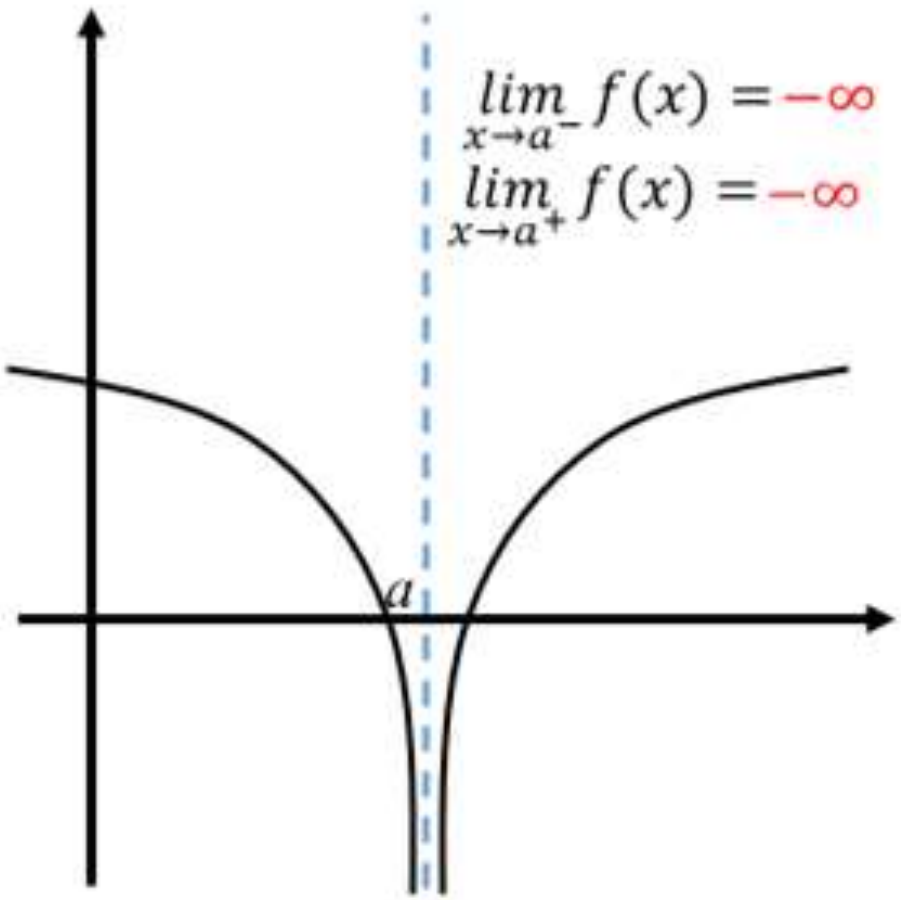
$$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x)$$

$$\lim_{x \to a} f(x) = \nexists$$

Limit at a point

Example 4



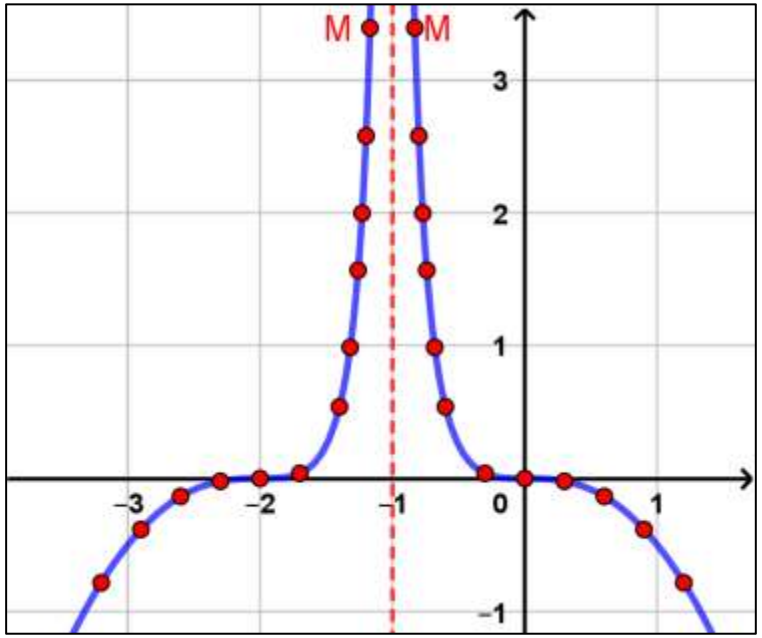


$$\lim_{x\to a} f(x) = \nexists$$

$$\lim_{x\to a} f(x) = -\infty$$

Vertical asymptote

If a point M of the curve moves away indefinitely while approaching toward a vertical line of equation $x = a$, this line is called **vertical asymptote** of the the curve.



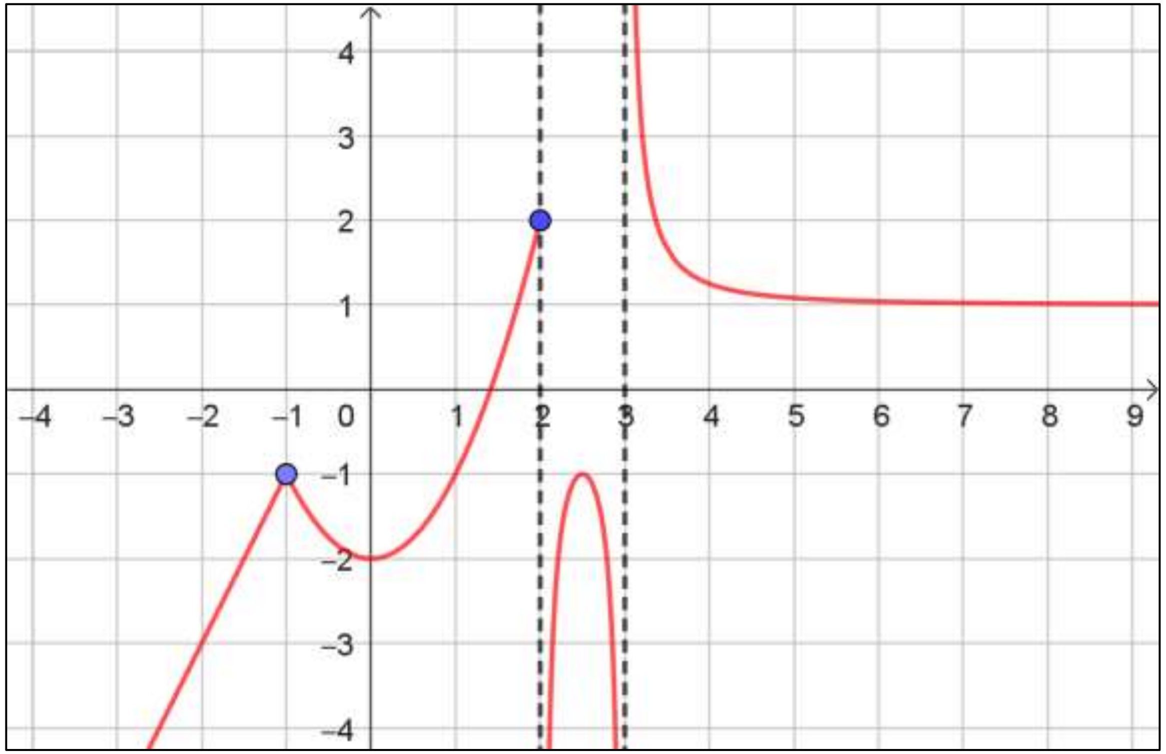
In this case:

$$\lim_{x \to a} f(x) = \pm\infty$$

Remark: the function f is not defined at x=a.

Application

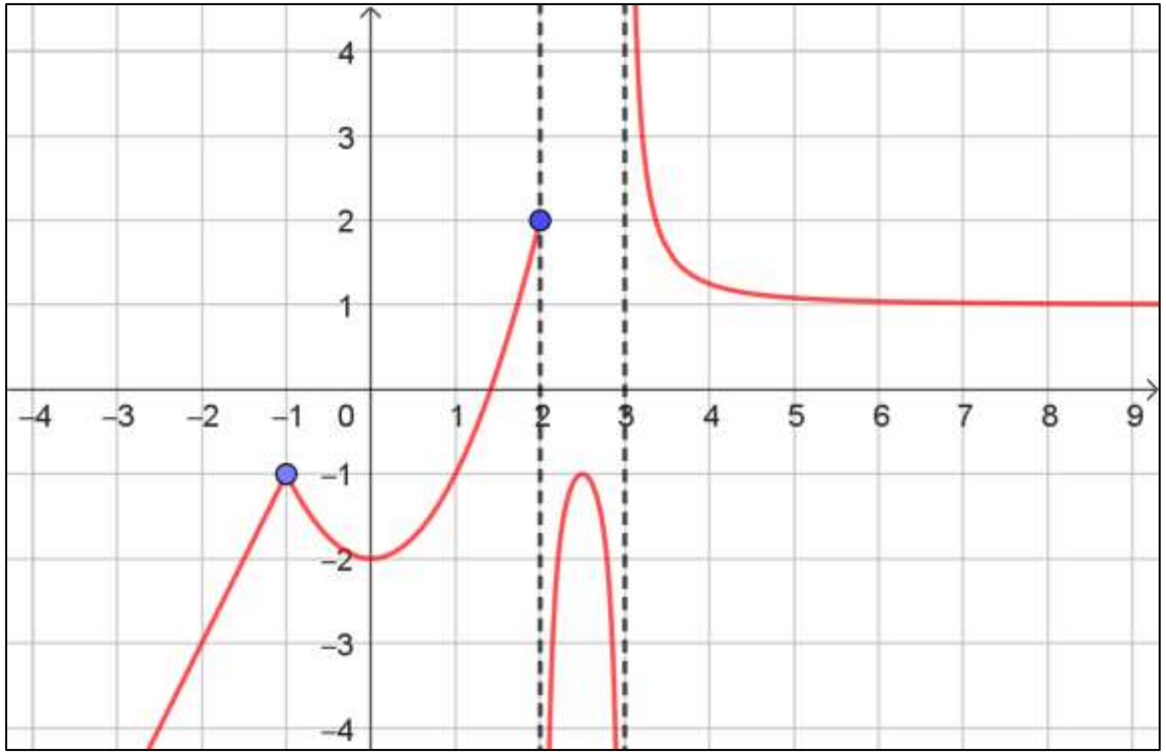
Find the limits in each case and determine the vertical asymptote if exists.



$$\lim_{x \to -1} f(x) = -1$$

Application

Find the limits in each case and determine the vertical asymptote if exists.



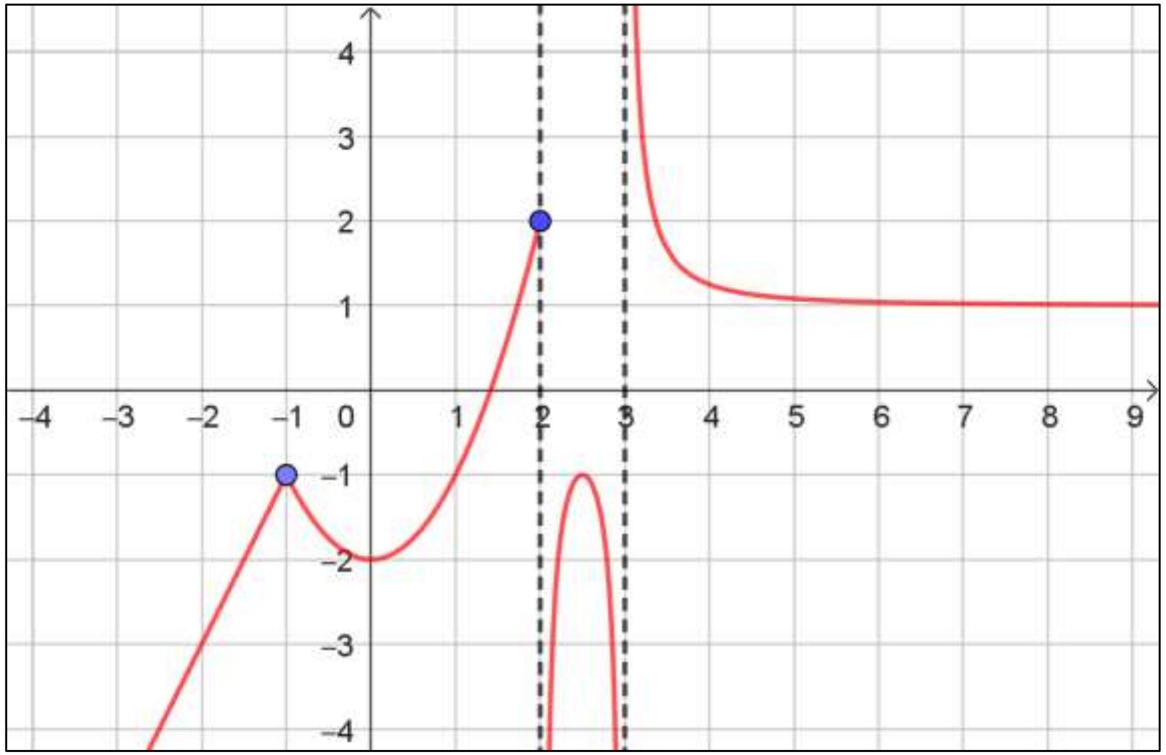
$$\lim_{x\to 2} f(x) = \nexists$$

Since $\lim_{x\to 2^-} f(x) = 2$ and

$\lim_{x\to 2^+} f(x) = -\infty$

Application

Find the limits in each case and determine the vertical asymptote if exists.

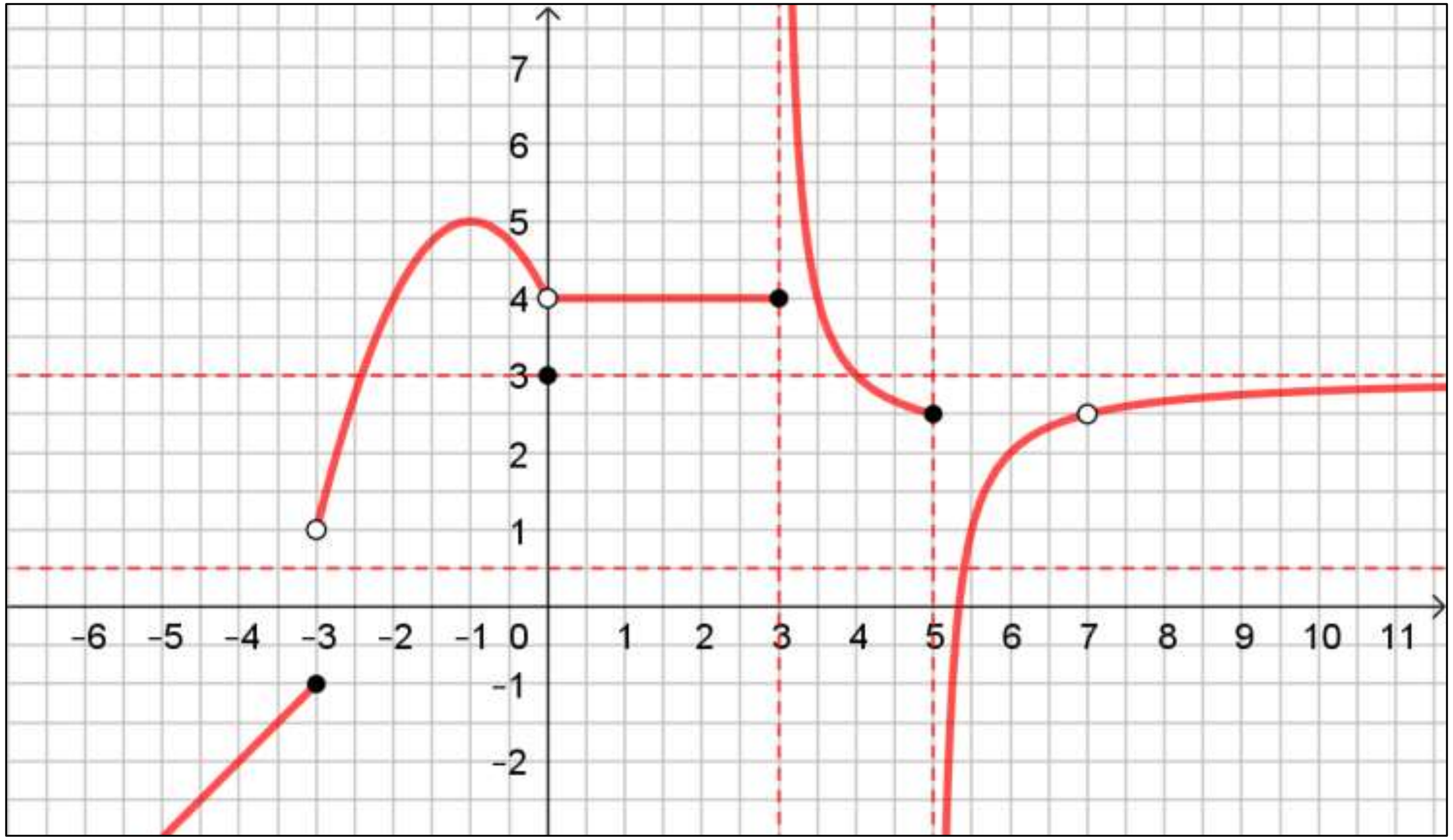


$\lim_{x \to 3} f(x) = \nexists$

Since $\lim_{x \to 3^-} f(x) = -\infty$ and

$\lim_{x \to 3^+} f(x) = +\infty$

Time for practice



$\lim_{x \to -\infty} f(x) = -\infty$

$\lim_{x \to +\infty} f(x) = 3$

$\lim_{x \to -3^-} f(x) = -1$

$\lim_{x \to -3^+} f(x) = 1$

$\lim_{x \to -3} f(x) = \nexists$

$\lim_{x \to 0^-} f(x) = 4$

$\lim_{x \to 0^+} f(x) = 4$

$\lim_{x \to 0} f(x) = 4$

$\lim_{x \to 3^-} f(x) = 4$

$\lim_{x \to 3^+} f(x) = +\infty$

$\lim_{x \to 3} f(x) = \nexists$

$\lim_{x \to 5^-} f(x) = 2.5$

$\lim_{x \to 5^+} f(x) = -\infty$

$\lim_{x \to 5} f(x) = \nexists$